

More Orbital Motion

$$a_{\text{centripetal}} = a_{\text{grav}}$$

$$\frac{v^2}{r} = \frac{GM}{r^2}$$

$$v = \sqrt{\frac{GM}{r}}$$

$$g = 9.8 \text{ m/s}^2$$

$$= \frac{GM}{R_e^2}$$

$$g = \frac{GM}{(R_e + h)^2}$$

$$g = \frac{GM}{r^2}$$

Geosynchronous Satellite

$$T = 1 \text{ day}$$

$$r = ?$$

orbital period
= Earth's rotational period

$$T = \frac{2\pi r}{v}$$

$$v = \sqrt{\frac{GM}{r}}$$

$$= 2\pi r \sqrt{\frac{r}{GM}}$$

$$= \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

$$\frac{\sqrt{GM}}{2\pi} T = r^{3/2}$$

$$\left(\frac{GM}{2\pi}\right)^{2/3} T^{2/3} = r$$

$$\frac{(GM)^{2/3}}{(2\pi)^{2/3}} T^{2/3} = r$$

$$\left(6.67 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \times 5.99 \times 10^{24} \text{ kg}\right)^{2/3}$$

$$(6.28)^{2/3}$$

$$\times (24 \cdot 3600 \text{ s})^{2/3} = 4.23 \times 10^7 \text{ m}$$

$$\left[\left(\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \text{ kg}\right)^{2/3} \text{ s}^{2/3}\right] = [\text{m}] \text{ correct dimension } \checkmark$$