Momentum

$$\vec{F} = m\vec{a} \qquad \text{Newton's 2nd Law}$$

$$= m\frac{\Delta\vec{v}}{\Delta t}$$

$$= \frac{\Delta(m\vec{v})}{\Delta t}$$

$$= \frac{\Delta\vec{p}}{\Delta t} \qquad \text{where } \underbrace{\vec{p} \equiv m\vec{v}}_{\text{definition}}$$

$\vec{p}$ is the momentum of the particle with mass m and velocity $\vec{v}$.

$$\boxed{\vec{p} = m\vec{v}}$$

For more than one mass in system

$$\vec{p} = m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots$$

If there are no external forces on the system, its momentum is constant.

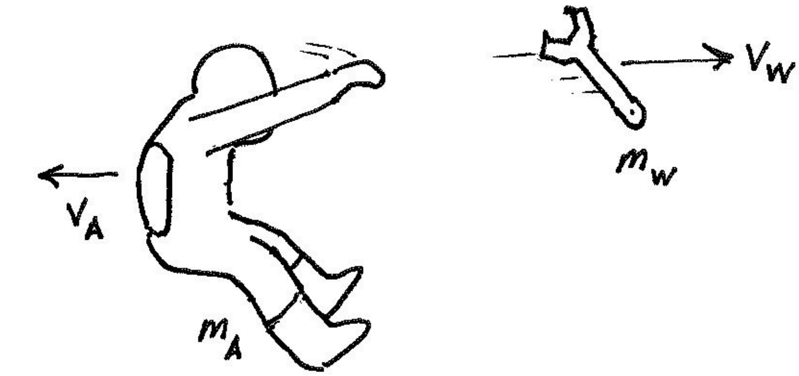
$$\boxed{\text{If } F=0,\ p=\text{constant}}$$

we say that "p is <u>conserved</u>".

Isolated system – system on which no external forces act.

→ Momentum of an isolated system is conserved.

Example: An astronaut throws a wrench.



m_A = mass of astronaut
V_A = velocity of astronaut
m_W = mass of wrench
V_W = velocity of wrench

Before she throws

$$V_{A_{Before}} = 0 \qquad V_{W_{Before}} = 0$$

Total momentum before

$$P_{Before} = m_A \underbrace{V_{A_{Before}}}_{0} + m_W \underbrace{V_{W_{Before}}}_{0} = 0$$

After she throws

$$P_{After} = m_A V_{A_{After}} + m_W V_{W_{After}} = 0$$

Because momentum of system is conserved. i.e. $P_{After} = P_{Before}$

Can solve for $V_{A_{After}}$.

$$V_{A_{After}} = \frac{-m_W V_{W_{after}}}{m_A}.$$

Let $m_A = 100\text{kg}$, $V_{W_{After}} = 10\text{ m/s}$, $m_W = 1\text{kg}$

$$V_{A_{After}} = \frac{-(1\text{kg})(10\text{ m/s})}{100\text{ kg}} = 0.1\text{ m/s}$$

Collisions

Elastic — No kinetic energy lost

Inelastic — kinetic energy lost

[Perfectly or Completely Inelastic — Final velocities of objects is the same (e.g. they stick together)]

Impulse

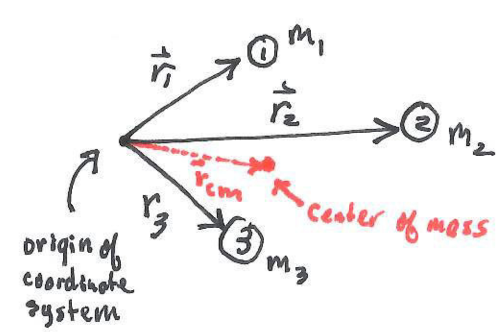
Have $\vec{F} = \frac{\Delta \vec{P}}{\Delta t} \Rightarrow \underbrace{\vec{F}\Delta t}_{\text{impulse}} = \Delta \vec{P}$

Impulse – (the force acting on an object) x (the time that the force acts)

The impulse on an object is the change in its momentum.

Center of Mass (of a multi-particle system)

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$



Note: No particle is necessarily present at the center of mass

$$\vec{V}_{cm} = \frac{m_1\vec{V}_1 + m_2\vec{V}_2 + m_3\vec{V}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

so

$$(m_1 + m_2 + m_3 + \cdots)\vec{V}_{cm} = m_1\vec{V}_1 + m_2\vec{V}_2 + m_3\vec{V}_3 + \cdots$$

$$(\text{total mass})\,\vec{V}_{cm} = (\text{total momentum})$$

$$\boxed{M_T \vec{V}_{cm} = P_T}$$