Stress & Strain

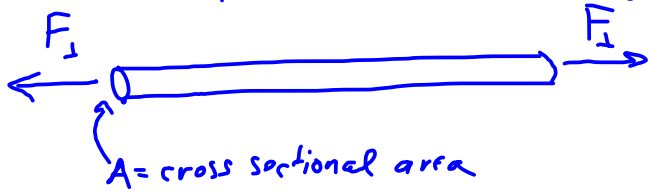
Hooke's Law = $F = -kx$

Generalized Hooke's Law

Case 1: Linear compression/expansion

stress = -(constant) x strain

$\text{stress} = \frac{F_\perp}{A}$ $\quad \text{strain} = \frac{\Delta \ell}{\ell_0}$



ℓ_0 = initial length

ℓ = length after force applied

$$\Delta p = -Y \frac{\Delta \ell}{\ell_0}$$

Δp = stress $\quad \frac{\Delta \ell}{\ell_0}$ = strain

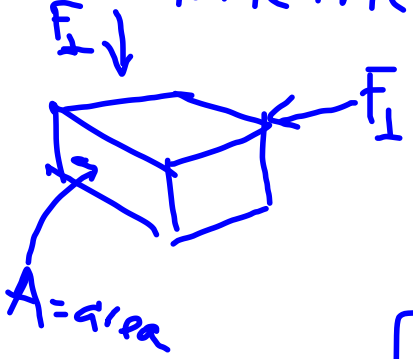
Y = Young's modulus > 0

stretching force $\Rightarrow$ "tensile" stress
" strain

compressing force $\Rightarrow$ compressive stress
" strain

Generalized Hooke's Law Case 2

Volumetric compression/expansion



$$\text{stress} = \frac{F_\perp}{A}$$

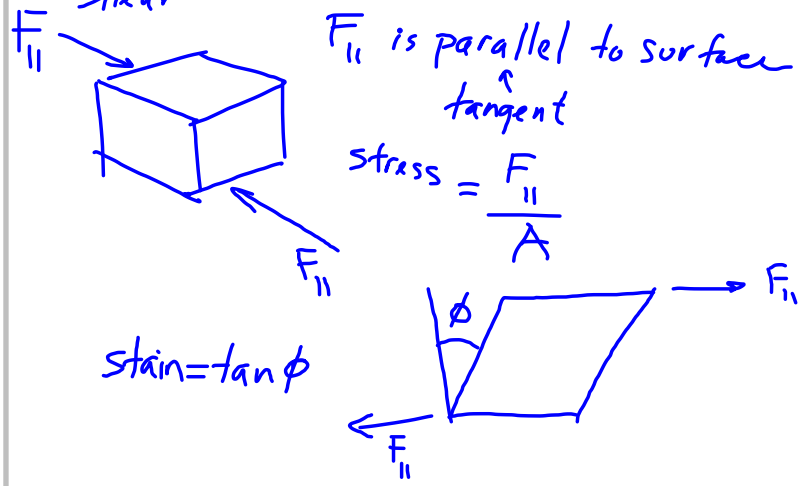
$$\text{strain} = \frac{\Delta V}{V_0}$$

$$\boxed{\Delta p = -B\frac{\Delta V}{V_0}}$$

Δp = stress $\quad \frac{\Delta V}{V_0}$ = strain

B = Bulk modulus > 0

Shear



$F_{\parallel}$ is parallel to surface
↑ tangent

$$\text{stress} = \frac{F_{\parallel}}{A}$$

$$\text{strain} = \tan\phi$$

$$\text{stress} = (\text{constant}) \times \text{strain}$$

$$\frac{F_{\parallel}}{A} = -S \tan\phi$$

$$\frac{F_{\parallel}}{A} \approx -S\phi \qquad \text{when } \phi \ll 1 \text{ rad}, \ \tan\phi = \phi$$

↑ Shear Modulus

elasticity – returns after forces removed (rubber) eg.

plasticity – retains new shape after forces removed (taffy) eg

Unit of Y and B = Pascal = $\frac{N}{m^2}$