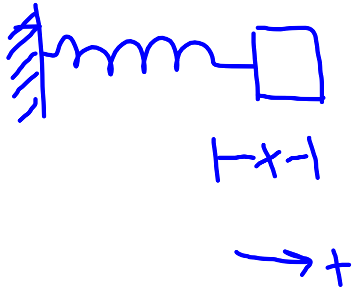
Simple Harmonic Motion

Due to a force of the following type

$$F = -kx$$

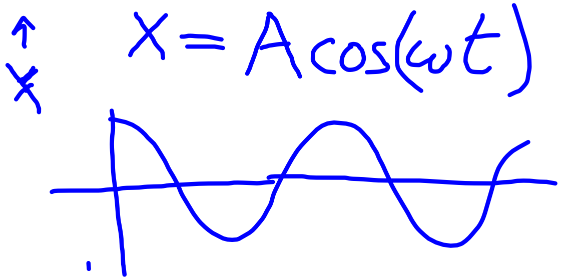
displacement from equilibrium



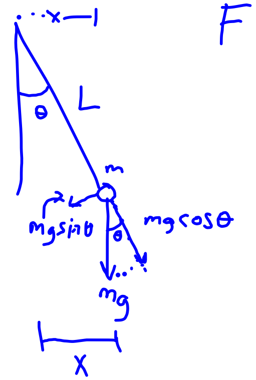
$F = -kx^2$ will **NOT** give simple harmonic motion

Simple harmonic motion is sinusoidal,

$$x = A\cos(\omega t) \qquad \omega = 2\pi f$$



Pendulum



$$F = -mg\sin\theta$$
$$= -mg\theta \qquad \theta \ll 1$$
$$= -mg\frac{x}{L}$$
$$= -\frac{mg}{L}x$$
$$= -kx$$

Harmonic Oscillator - a system that undergoes simple harmonic motion.

$$\omega_{pendulum} = \sqrt{\frac{g}{L}} \qquad \omega = 2\pi f$$

$$\omega_{spring} = \sqrt{\frac{k}{m}}$$

period $= T$

$$T_{pendulum} = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

$$T_{spring} = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

Simple Harmonic motion with frequency ω is the projection ("shadow") of circular motion with angular velocity ω.

ω is called the circular frequency.

Damping

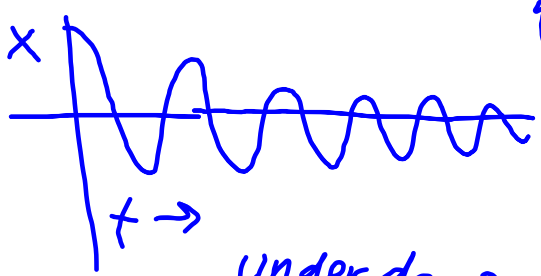
Friction

$$v = \frac{dx}{dt} = \frac{\Delta x}{\Delta t}$$

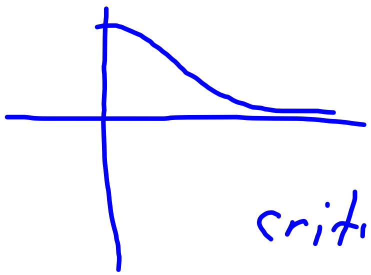
$$F = -kx - \alpha v$$

damping force

α = damping coef



underdamped



critically damped = reaches static equilibrium in least amount of time

overdamped

Resonance

The efficient absorption of energy by a oscillator from an oscillating force whose frequency is the same as that of the oscillator.

Energy of Harmonic Oscillator

$$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

Potential energy

Kinetic energy

- When oscillator is a max amplitude of oscillation, energy is 100% potential.
- When displacement is zero, energy is 100% kinetic

Mechanical Waves

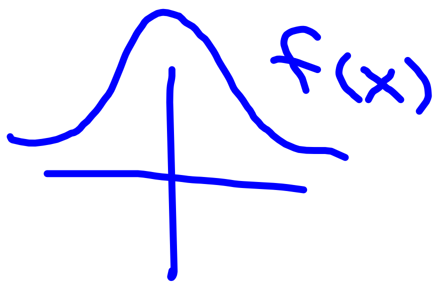
- A wave that propagates by displacing a medium.
- example: ripples in a pond
  (medium is water)
- example: sound moves by displacing air

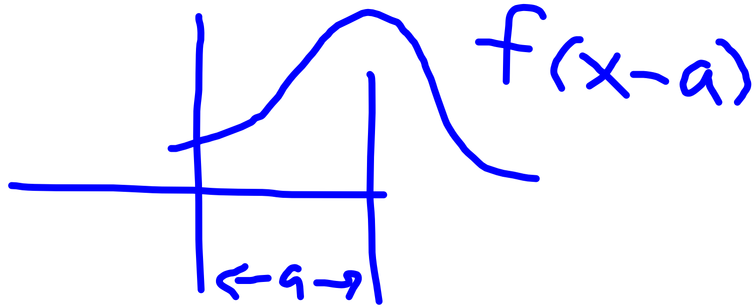
Two types of mechanical waves

1) Transverse – medium displaced in direction perpendicular to that of wave progation. (e.g. ripples in pond)
2) Longitudinal – medium displaced in direction parallel to that of wave propagation.
   (e.g. "slinky wave" or sound)

Math Description of a wave

$$y(x,t) = f(x-vt)$$





$$a = vt$$

$f(x-vt)$ moves rightward with speed v.

Principle of Superposition

- Only allowed waves are solutions to the wave equation (application of Newton's 2nd law to the medium)

  $y(x,t) = f(x - vt)$

- If $y_1(x,t)$ is a solution AND $y_2(x,t)$ is a solution THEN

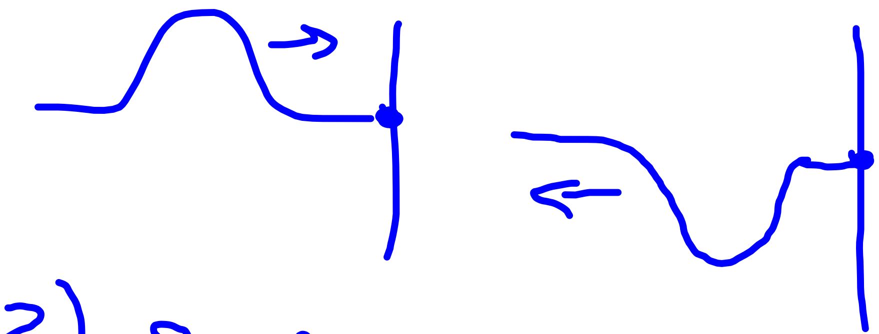
  $y(x,t) = y_1(x,t) + y_2(x,t)$

  is also a solution.

i.e. WAVES ADD

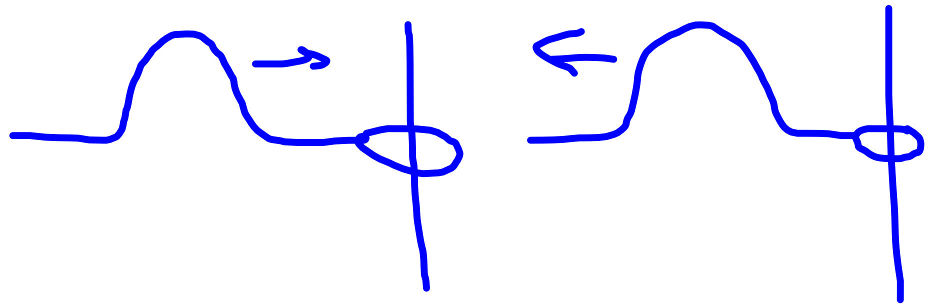
Reflections of a pulse

1) String has fixed end



Reflected pulse is inverted

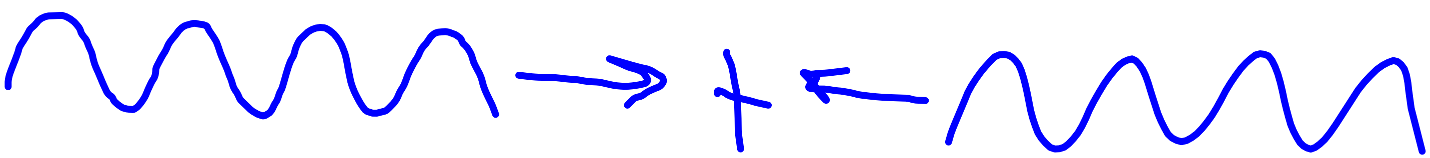
2) String has a free end



Reflected pulse is not inverted

Standing waves

The sum of identical periodic waves that travel in opposite directions.

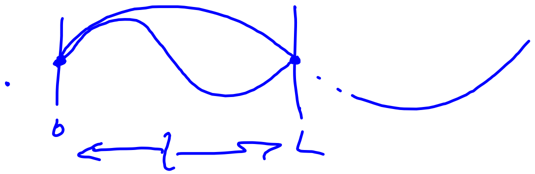


$$y(x,t) = A\sin k(x-vt)$$
$$+ A\sin k(x+vt)$$
$$= A\sin kx\cos\omega t - A\cos kx\sin\omega t$$
$$+ A\sin kx\cos\omega t + A\cos kx\sin\omega t$$
$$= (2A\cos\omega t)\sin kx$$

No vt

Needed $\sin(a+b) = \sin a \sin b$
$+\cos a \sin b$

$$k = \frac{2\pi}{\lambda} \qquad kv = \frac{2\pi}{\lambda} \lambda f = 2\pi f = \omega$$



$(\ )\sin kL = 0$

$(2A\cos\omega t)\sin kL = 0 \Rightarrow kL = n\pi$

$\frac{2\pi}{\lambda}L = n\pi$

$\frac{2L}{\lambda} = n$

$\lambda_n = \frac{2L}{n}$

$\frac{\lambda_1}{2} = L$

$\lambda_2 = L$

Fundamental frequency

lowest allowed freq.

$f_n = \frac{v}{\lambda_n} = \frac{v}{2L}n$

$\lambda f = v$

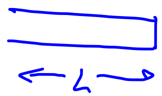
$f = \frac{v}{\lambda}$

f_1 = fundamental

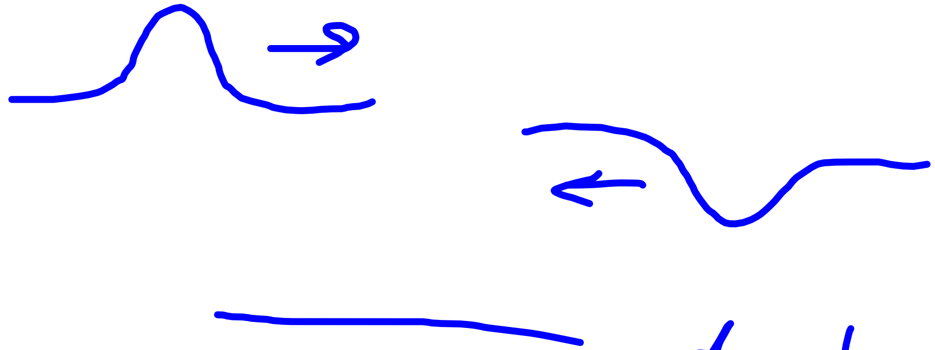
$f_{n>1}$ = harmonic

For wind instrument
(with one open end)

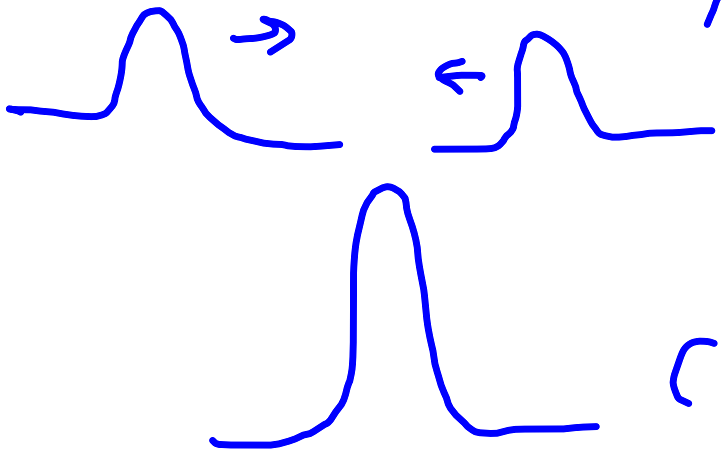
$f_n = \frac{nv}{4L}$



Interference

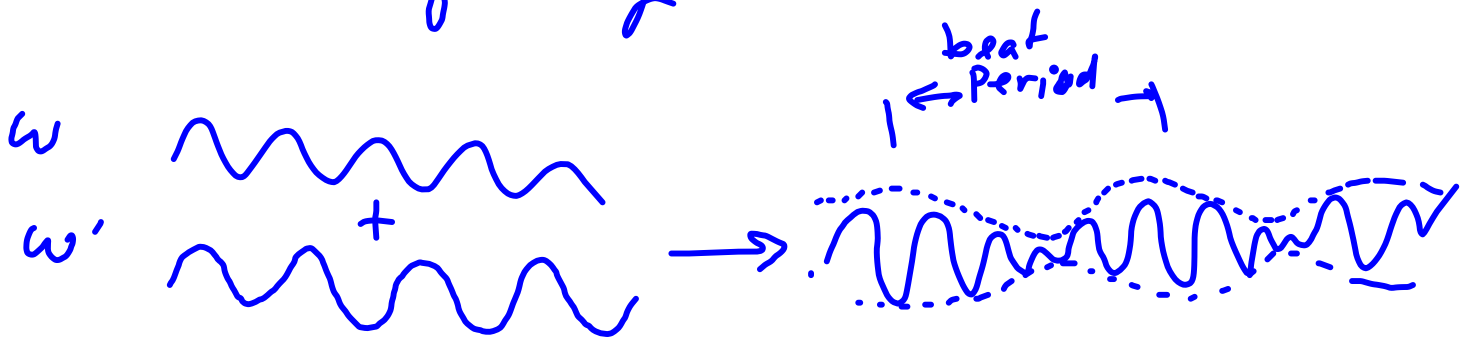


destructive interference



constructive interference

Beats – Amplitude variation of the sum of two periodic waves of slightly different frequency



frequency of beat

= freq of wave 1 − frequency of wave 2

$$f_{beat} = f_1 - f_2$$

Doppler Effect

- Change in perceived frequency of a sound due to the motion of the emitter or listener

$$f_L = \frac{v + v_L}{v + v_S} f_S$$

v = velocity of sound
v_L = velocity of listener
v_S = velocity of source
f_S = frequency of source
f_L = frequency heard by listener

$v_S < 0$
$\Rightarrow$ source moves toward listener

$v_L > 0$
listener moving toward source

Speed of sound

in air = 330 m/s = 760 mph

in human body = 1540 m/s

$$v = \sqrt{\frac{S}{\rho}}$$

S = measure of "stiffness"
(young modulus,
Bulk modulus
tension in string)

ρ = density

Doppler Effect for light

$$f_L = \sqrt{\frac{c-v}{c+v}}\, f_s$$

$v<0$ if observer moves toward source