# FLUID MECHANICS

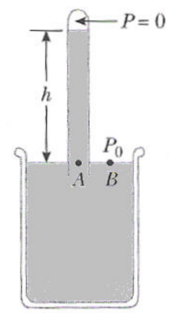
The study of the properties of fluids resulting from the action forces.

Fluid – a liquid, gas, or plasma

We will only consider incompressible fluids – i.e. liquids

**Pressure**

$$P = \frac{F_{\perp}}{A}$$ = (normal force) / (area on which force acts)




Gauge Pressure = $P - P_{atmosphere}$

Barometer – Fluid rises until pressure at *A,* due its weight, equals atmospheric pressure at *B.*

Unit: mm Hg (millimeters that mercury rises)

Unit: Newtons/$m^2$ = Pascal

Unit: mm $H_2O$ (millimeters that water rises)

1 atmosphere = 760 mm Hg = 101,325 Pascals

1 mm Hg = 13.6 mm $H_2O$

**Pascal's Law**

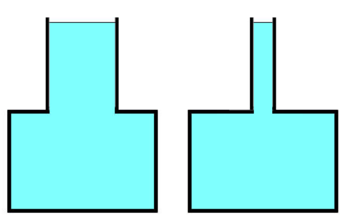
-- For a static fluid within a container, the pressure at any point depends solely on its vertical position.

-- The difference in pressure between two points depends solely on their vertical separation

$$\Delta P = \rho g \Delta h$$

$\Delta P$ = pressure difference
$\rho$ = fluid density
$g$ = acceleration of gravity
$\Delta h$ = vertical separation

Q. In which of the following containers of water is the pressure greatest in the center of the lower compartment?

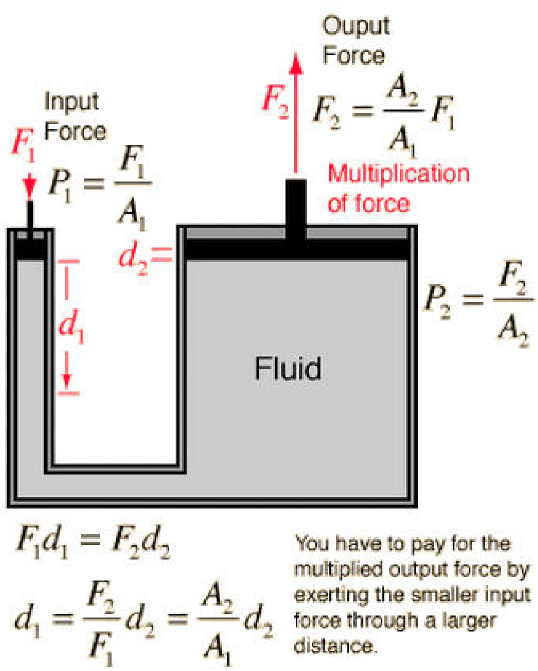


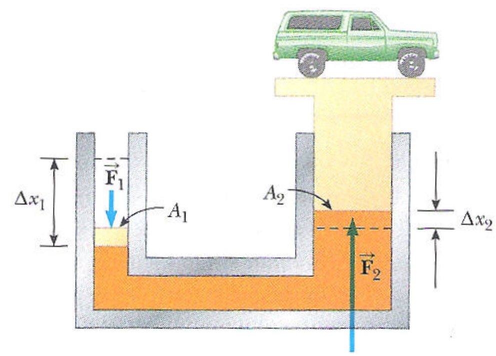
A. Pressures are the same in both.

Q. A swimming pool is 4 meters deep. What is the pressure at a) the top of the pool and b) the bottom?

A. a) pressure at top = $P_{atmosphere}$ = $10^5$ Pascals
b) pressure at bottom = $P_{atmosphere} + \Delta P$
= $10^5$ Pascals + (1000 kg/m$^3$)(9.8 m/s$^2$)(4 m)
= $1.4 \times 10^5$ Pascals

Application: Hydraulic Force Amplification




**Pascal's Law ➔** $P_1 = P_2$ ➔ $\frac{F_1}{A_1} = \frac{F_2}{A_2}$ ➔ $F_2 = \frac{A_2}{A_1} F_1$

Force $F_1$ amplified by factor $A_2/A_1$.

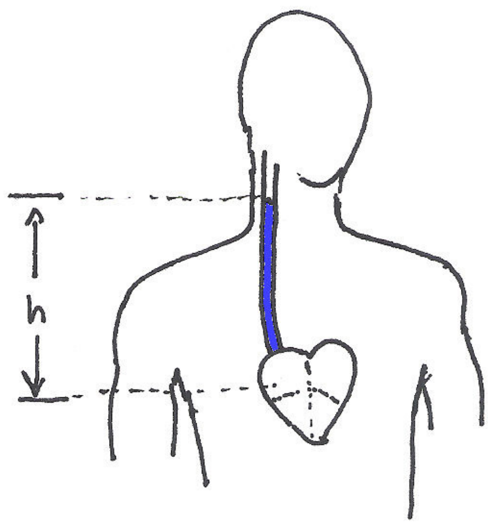
Q. Could an old woman who can exert a maximum force of 5 lbs, lift a 68-ton Abrams tank, if $A_1$ is a disk of radius 0.1 m and $A_2$ is a disk of radius 20 m?

A. $F_2 = \frac{A_2}{A_1} F_1 = \frac{\pi r_2^2}{\pi r_1^2} 5lbs = \left(\frac{20}{0.1}\right)^2 5lbs = 40{,}000 \times 5lbs = 200{,}000 lbs$ = 100 tons

**YES.**

This is how a slight touch of your brake pedal can stop your speeding car.

Medical Application: <u>Finding Pressure in Vena Cava</u> (without surgery)




$h$ = distance from right atrium of heart to highest visible bulge of jugular vein

Jugular vein is a tube going straight up from right atrium.

So $P_{\text{Rt. Atrium}}$ in mm $H_2O$ = $h$. $(\rho_{blood} \approx \rho_{water})$

Right atrium connected to Vena Cava ➔ $P_{\text{Rt. Atrium}} = P_{\text{Vena Cava}}$

To convert to mm Hg, divide by 13.6.

Example: $h$ = 109 mm. Then

$P_{\text{Vena Cava}}$ = 109 mm $H_2O$ = 109 ($13.6^{-1}$ mm Hg)

= 8 mm Hg (high end of normal range)

[$P_{\text{Vena Cava}}$ is called "Central Venous Pressure"]

**Achimedes' Principle**

-- Buoyant force on an object immersed in a fluid is the weight of the fluid that it displaces

Buoyant force – upward force on an object immersed in a fluid

Q. Which is greater the buoyant force on a cube of iron, or that on a cube of Styrofoam of the same size?

A. Buoyant force is the same for both. [Because they're the same size, they displace the same amount of fluid.]

<u>Floating</u>

To float, upward force on object must exceed downward force.


$F_{Buoyant} > F_{gravity}$

(weight of water displaced by object) > (weight of object)

$$\rho_{fluid} V g > \rho_{object} V g \;\rightarrow\; \rho_{fluid} > \rho_{object}$$

[This is why its easy to float in the Great Salt Lake or in the Dead Sea.]

Let fluid = water. To float

$$\rho_{water} > \rho_{object} \;\rightarrow\; 1 > \frac{\rho_{object}}{\rho_{water}} \;\rightarrow\; \frac{\rho_{object}}{\rho_{water}} < 1$$

i.e. the "specific gravity" of the object must less than 1

**specific gravity** of object = $\frac{\rho_{object}}{\rho_{water}}$

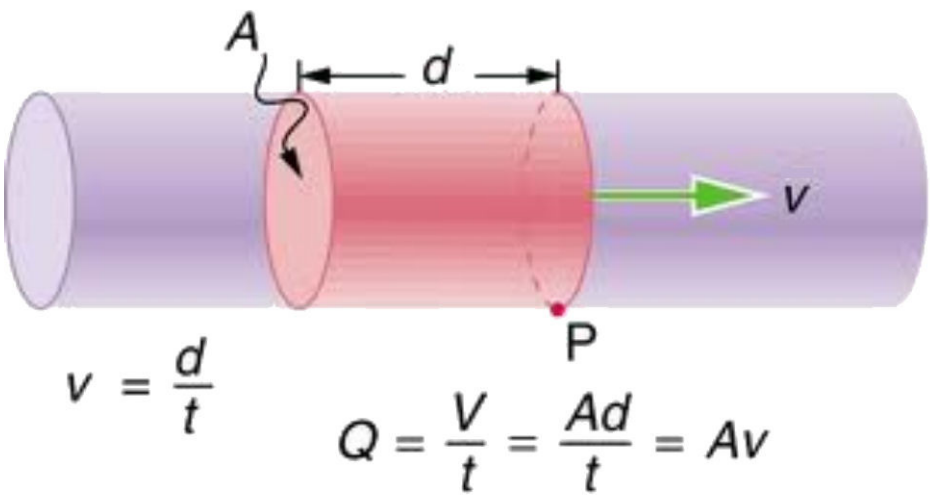
Human brain has neutral buoyancy ($F_{Buoyant} = F_{gravity}$), which prevents it from resting on the bottom of the skull or being pinned at the top of it.

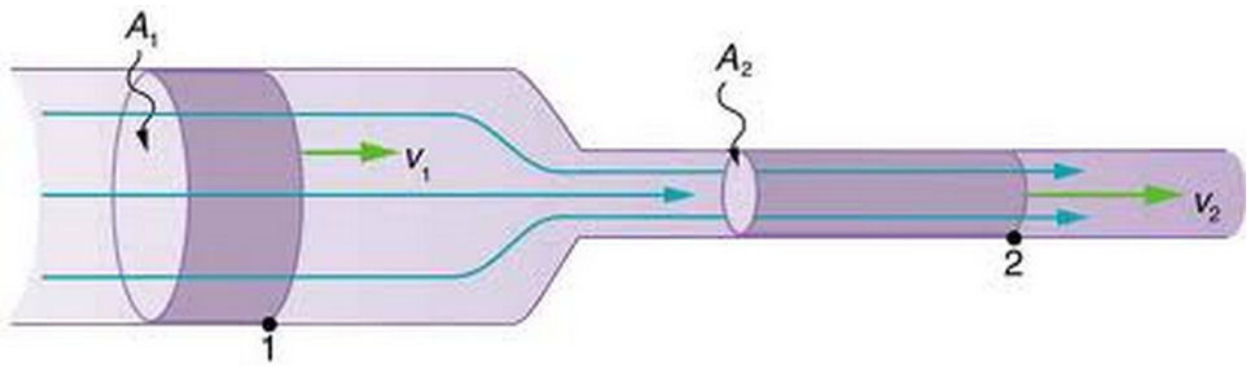
i.e. $\rho_{brain} \approx \rho_{cerebro-spinalfluid}$

**Continuity of Fluid Flow**

For fluid flowing in a tube, whose diameter varies, the flow rate is the same everywhere. (Continuity assumption)

Flow rate = (volume of fluid that flows past a point) / (time it took to flow past)




Continuity ➔ Flow Rate at 1 = Flow Rate at 2

$$Q_1 = Q_2$$

$$A_1 v_1 = A_2 v_2$$

Notice $v_2 = \frac{A_1}{A_2} v_1$

*The more the tube (or artery) narrows, the faster fluid velocity* (even the flow rate remains constant)

Q. Blood travels at 30 cm/s in an aorta, whose radius is 1.5 cm. Blood travels at 0.1 cm/s in capillaries whose radii are 5 x $10^{-4}$ cm. About how many capillaries are in the human body?

A. Flow rate must be constant

(Flow rate in aorta) = (Combined flow rate of all capillaries)

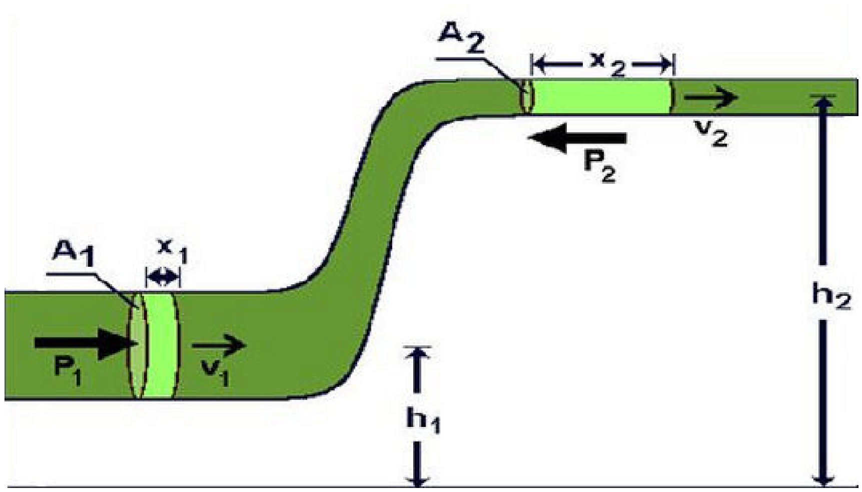
= N x (Flow rate in 1 capillary)

$$A_a v_a = N A_c v_c$$

$$N = \frac{A_a v_a}{A_c v_c} = \frac{\pi(1.5)^2 \times 30}{\pi(5\times10^{-4})^2 \times 0.1} = 2.7\times10^9$$

2.7 billion capillaries

**Bernoulli Equation**




$$P_1 + \tfrac{1}{2}\rho v_1{}^2 + \rho g h_1 = P_2 + \tfrac{1}{2}\rho v_2{}^2 + \rho g h_2$$

i.e. $$P + \tfrac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

Follows from energy conservation.

Note: For constant *h, an increase in fluid velocity requires a decrease in* *pressure*.

Medical Consequence: Vascular Flutter

1. Section of artery narrowed by plaque
2. Blood velocity increases. (Continuity)
3. Pressure in artery section decreases. (Bernoulli equation)
4. Pressure from tissue exterior to artery section exceeds pressure within section.
5. Exterior pressure collapses artery section.
6. Blood velocity goes to zero.
7. Pressure increases beyond exterior pressure again. (Bernoulli eq)
8. Blood velocity increases.
9. Sequence repeats.

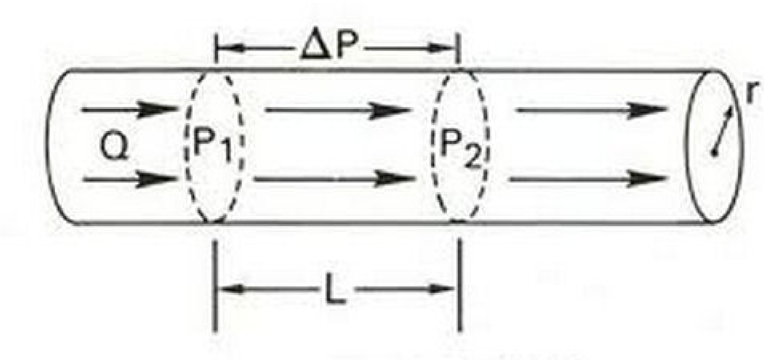
Partially Explains Aerodynamic Lift.

Explains Attraction Between Closely Passing Ships

**Poiseuille's Law**

Describes change in pressure between ends of a tube due to fluid viscosity.

viscosity = "thickness" (water has low viscosity; molasses has high viscosity)




$$\Delta P = \frac{8\eta L}{\pi r^4} Q$$

$Q$ = volume flow rate
$\eta$ = viscosity (Pascal - seconds)
$r$ = radius of tube
$L$ = length of tube section

Often written

$\Delta P = RQ$ , 🡐 Central equation of Hemodynamics where $R = \frac{8\eta L}{\pi r^4}$ is called the fluid *resistance*

Resistance rapidly increases, when the radius decreases.
Resistance increase with viscosity (e.g. sickle cell disease)

**Turbulence**

Deviation from Laminar flow

Laminar flow – paths of particles in fluid do not cross or reverse direction

Reynolds number (Re) – a quantifier of turbulence

$$\mathrm{Re} = \frac{\rho v d}{\eta}$$ (Don't bother to memorize this)

= (density X velocity X pipe diameter) / viscosity

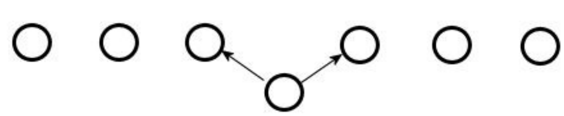
Can express in terms of flow rate Q

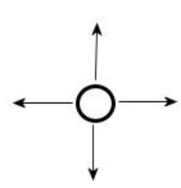
$$\mathrm{Re} \approx \frac{\rho Q}{d\eta}$$ (Don't bother to memorize this)

Turbulence
increases with increasing density or flow rate
decreases with increasing artery/pipe diameter or blood viscosity

causes arteries to audibly vibrate.
Audible turbulence within the heart is called a "heart murmur"
Audible turbulence within an artery is called a "bruit"

**Surface Tension**



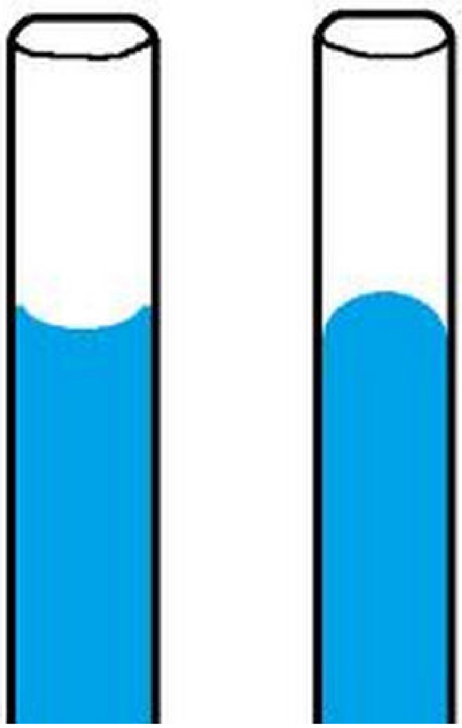


Deformation of surface causes upward pressure change

$\Delta P = \frac{2\gamma}{R}$ R = radius of curvature of deformation (depression)

$\gamma$ = surface tension (energy/area)

Meniscus



concave convex

convex ➔ fluid cohesion > fluid adhesion (creeps downward)

concave ➔ fluid adhesion > fluid cohesion (creeps upward)